X-ray Microtomography Study of a Model Crumpled Membrane

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Introduction

Both in materials research [1] and in string theory [2], extensive recent theoretical work has focused on the equilibrium phase diagrams of tethered manifolds. In addition there has been a surge of interest in a special case of nonequilibrium tethered (2-d) membranes, i.e. the problem of crumpled membranes. Recent work in this field includes:

- a first-principles understanding of the shape and elastic energy of creases [3].
- numerical and theoretical evidence for a characteristic length scale in the time dynamics of Euler bifurcation of sheets [4].
- the discovery of 'stress condensation' in crumpled membranes [5].
- spatial correlation of the 'scar' pattern on 'unfolded' crumpled membranes [6].
- power-law scaling of acoustic noise from the compression of crumpled membranes [7].

A central theme in this ensemble of studies is the importance of both the details of the distribution of elastic energy and the details of the mechanisms for dissipation. Additionally, in many cases the relevant structural correlation function to describe crumpling is not the mass-mass pair correlation function, but instead the joint probability density function for path lengths along the manifold relative to displacements in the embedding into Euclidean space. Hence, a strong interaction between theory and experiment will require real-space structural information about crumpled membranes.

Here, we investigate the use of x-ray microtomography (XMT) in the study of (nonequilibrium) mechanically crumpled membranes.

Methods and Materials

The experiment was performed at sector 20-ID of the Pacific Northwest Consortium (PNC) beamlines at the Advanced Photon Source. The area detector of our prototype tomography apparatus follows the general considerations of Koch, *et al*., [8], with the exception that an inexpensive 8-bit room temperature CCD camera was used as an initial cost-saving measure. A cooled CCD camera will be used in the final apparatus. The tomography sample stage consisted of two miniature linear translators for centering mounted atop an Aerotech ART-50 rotary stage. The rotary stage was itself mounted atop a homemade two-axis motorized tilt stage; these two degrees of freedom were used to ensure that the rotation axis was simultaneously perpendicular to the beam direction and to the effective CCD line scan direction with precision 10^{-5} radians. A standard filtered backprojection algorithm was used to reconstruct the tomographs from the rotational sequence of radiograms.

The sample used in the measurements discussed here was manually crumpled into a spheroid from a 6-cm2 area of standard aluminum foil (Reynolds, USA). A photon energy of 20.0 keV was used.

Results

An XMT slice near the maximum cross-section of the spheroid is shown in Figure 1. The lateral space across the figure is approximately 2.5mm. Our sample stage alignment was sufficiently precise that the tomographs from successive lines on the CCD detector are statistically independent. Thus, we have performed a fully 3-dimensional characterization of the mass distribution of a crumpled membrane.

Fig. 1. An XMT slice of a crumpled membrane.

Discussion

Our initial analysis of several high-order structural correlation functions of the crumpled membrane has revealed a well-defined characteristic length associated with the linear connectivity of the void phase. These results will be presented in detail elsewhere [9] and are of immediate relevance for the modeling of the effective transport properties of membranous disordered materials [10], and for previous numerical and theoretical work on the dynamics of Euler bucking [4]. With the improved signal to noise from a CCD camera upgrade, we anticipate being able to calculate the high-order structural correlation functions that appear in the theory of equilibrium crumpling. The consequent comparison of the equilibrium theory and the nonequilibrium experiment will elucidate the role of dissipation in this problem and also the effective degree of ergodicity of the driven nonequilibrium system.

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